

Unit 16
Exponentials and Logarithms
Day 1
The Exponential Function

Recall the following property of exponents:

$$a^{\frac{m}{n}} = \left(\sqrt[n]{a^m} \right)$$

example:

$$32^{\frac{2}{5}} = \sqrt[5]{32^2}$$

Additional Properties of Exponents:

For any real number $a > 0$, $a \neq 1$, and any real number x , the following statements are true.

(a) a^x is a unique real number.

(b) $a^b = a^c$ if and only if $b = c$.

(c) If $a > 1$ and $m < n$, then $a^m < a^n$.

(d) If $0 < a < 1$ and $m < n$, then $a^m > a^n$.
 $\frac{1}{4} > \frac{1}{8}$ $\left(\frac{1}{2}\right)^2 > \left(\frac{1}{2}\right)^3$

Solving Exponential Equations

1)

$$\begin{aligned} 9^x &= 3 \\ (3^2)^x &= 3 \\ 3^{2x} &= 3^1 \\ 2x &= 1 \\ x &= \frac{1}{2} \end{aligned}$$

2)

$$\begin{aligned} \left(\frac{1}{5}\right)^x &= 125 \\ 5^{-x} &= 5^3 \\ -x &= 3 \\ x &= -3 \end{aligned}$$

$$\begin{aligned}
 3) \quad \frac{125}{64} &= y^{-3} \\
 \frac{5^3}{4^3} &= y^{-3} \\
 \left(\frac{5}{4}\right)^3 &= y^{-3} \\
 \left(\frac{4}{5}\right)^{-3} &= y^{-3} \\
 \frac{4}{5} &= y
 \end{aligned}$$

$$\begin{aligned}
 4) \quad 16^x &= 64 \\
 4^{2x} &= 4^3 \\
 2x &= 3 \\
 x &= \frac{3}{2}
 \end{aligned}$$

~~$$4^{2x} = 8^2$$~~

$$\begin{aligned}
 5) \quad 25^p &= 625 \\
 5^{2p} &= 5^4 \\
 2p &= 4 \\
 p &= 2
 \end{aligned}$$

$$\begin{aligned}
 6) \quad 100^{2+t} &= 1000 \\
 10^{4+2t} &= 10^3 \\
 4+2t &= 3 \\
 2t &= -1 \\
 t &= -\frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 7) \quad 4^{k+2} &= 32 \\
 2^{2k+4} &= 2^5 \\
 2k+4 &= 5 \\
 2k &= 1 \\
 k &= \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 8) \quad 25^{1-t} &= 5 \\
 5^{2-2t} &= 5 \\
 2-2t &= 1 \\
 -2t &= -1 \\
 t &= \frac{1}{2}
 \end{aligned}$$

$$\begin{aligned}
 9) \quad 256^x &= 64^{2-x} \\
 2^{8x} &= 2^{12-6x} \\
 8x &= 12-6x \\
 14x &= 12 \\
 x &= \frac{6}{7}
 \end{aligned}$$

$$\begin{aligned}
 10) \quad \left(\frac{1}{36}\right)^{2-r} &= 216^{3r} \\
 \left(\frac{1}{6^2}\right)^{2-r} &= 6^{9r} \\
 6^{2r-4} &= 6^{9r} \\
 2r-4 &= 9r \\
 -7r &= 4 \quad r = -\frac{4}{7}
 \end{aligned}$$

12)

$$\left(\frac{1}{4}\right)^{x+1} = \left(\frac{1}{2}\right)^{x+1}$$

$$\left(\frac{1}{2}\right)^{2x+2} = \left(\frac{1}{2}\right)^{x+1}$$

$$2x+2 = x+1$$

$$\boxed{x = -1}$$

13)

$$\left(\frac{3}{5}\right)^{-x} = \left(\frac{25}{9}\right)^{x-2}$$

$$\left(\frac{3}{5}\right)^{-x} = \left(\frac{5^2}{3^2}\right)^{x-2}$$

$$\left(\frac{5}{3}\right)^x = \left(\frac{5}{3}\right)^{2x-4}$$

$$x = 2x - 4$$

$$-x = -4$$

$$x = 4$$

Homework

Unit 16 Day 1

p. 370-371: 1-8, 49-62